STATE VARIABLE FEEDBACK DESIGN OF A CONTROL SYSTEM FOR A COUPLED-CORE REACTOR

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ABSTRACT

In this study a control system for a coupled-core one delayed neutron group point reactor model with linear independent temperature reactivity feedback and a quadratic controller is designed and analyzed using a state variable feedback design technique[1]. The problem is formulated in matrix notation, therefore, retaining the generality of the method. The desired system dynamics are specified in terms of the closed loop transfer function. For the case in point the desired response is characterized by second order dynamics with a damping ratio of .707 and a zero velocity error response to a ramp power demand input. Simulation studies using a digital time response program based on the 4th order Runge-Kutta method, show that the desired system dynamics are exactly realized by feeding back all the state variables through constant gain elements.

The control system design based on the linearized model was applied to the non-linear problem. Simulation studies show that the desired system dynamics are obtained for step demands in power up to 35% of the equilibrium value. For perturbations up to 50% of equilibrium value there is only a slight change in the system dynamics. Further, the step response to negative inputs is more heavily damped than for corresponding positive inputs. The step response becomes more oscillatory as the equilibrium power is increased beyond the design value and more damped as the power level is decreased below the design value. Parameter variation effects on the desired response is also considered.

CHAPTER I

PHYSICAL REACTOR MODEL

The purpose of this work is to investigate the applicability of state variable feedback control to the design of a control system for a coupled core one delayed neutron group point reactor model with linear independent temperature reactivity feedback. In this study the system to be considered consists of identical reactors operating at the same power level, with parameter values indicative of the Kiwi-type reactors being developed for space propulsion.[2]

The system is described by the following set of generalized differential equations:

$$1)\quad \overset{\circ}{n}_i = \left(\frac{\rho_i-\beta}{\ell_i}\right) n_i + \lambda_i c_i + D_{2i}\,\frac{n_{(i+1)}}{\ell_{(i+1)}}\,\delta_i^1 + D_{1i}\,\frac{n_{(i-1)}}{\ell_{(i-1)}}\,\delta_i^2,$$

$$2)\quad \overset{\circ}{c}_i = \left(\frac{\beta}{\ell_i}\right) n_i - \lambda_i c_i,$$

$$3)\quad \overset{\circ}{T}_i = K_i n_i - a_i T_i,$$

$$4)\quad \rho_i = \rho_{ie} - \alpha_i T_i, \qquad\qquad i = 1,2$$

where

n_i = neutron density or power in the i^{th} core;

ρ_i = total reactivity in the i^{th} core;

β = total delayed neutron fraction;

ℓ_i = neutron generation time;

λ_i = effective delayed neutron precursor decay constant;

c_i = delayed neutron precursor concentration in the i^{th} core;

T_i = temperature in the i^{th} core;

K_i = reciprocal heat capacity of the i^{th} core;

a_i = reciprocal time constant for heat loss from the i^{th} core;

α_i = temperature coefficient of reactivity for i^{th} core;

D_{12} = coupling coefficient which gives the reactivity contribution to core 2 due to neutron leakage from core 1;

D_{21} = coupling coefficient which gives the reactivity contribution to core 1 due to neutron leakage from core 2;

ρ_{ie} = externally applied reactivity; ρ_{2e} = 0 since only one core is being controlled;

δ_i^j = Kronecker delta; $\delta_i^j = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$

and dotted variables ($\mathring{n}$) indicates the time derivative ($\frac{dn}{dt}$) of that variable.

CHAPTER II

LINEARIZED MODEL

Before the state variable feedback control design technique can be applied, the system equations must be linearized. This is accomplished by taking small perturbations about a steady state equilibrium value. Following this procedure Eqs. 1 to 4 become

$$5)\quad \delta\dot{n}_i = \left(\frac{\rho_{i0} + \delta\rho_i - \beta}{\ell_i}\right)(n_{i0} + \delta n_i) + \lambda_i\,(c_{i0} + \delta c_i) + + \frac{D_{2i}\delta_i^1}{\ell_{(i+1)}}(n_{(i+1)0} + \delta n_{(i+1)}) + \frac{D_{1i}}{\ell_{(i-1)}}\delta_i^2\,(n_{(i-1)0} + \delta n_{(i-1)})$$

$$6)\quad \delta\dot{c}_i = \frac{\beta}{\ell_i}(n_{i0} + \delta n_i) - \lambda_i(c_{i0} + \delta c_i)$$

$$7)\quad \delta\dot{T}_i = K_i\,(n_{i0} + \delta n_i) - a_i\,(T_{i0} + \delta T_i)$$

$$8)\quad \rho_{i0} + \delta\rho_i = \rho_{ie0} + \delta\rho_{ie} - \alpha_i(T_{i0} + \delta T_i) \qquad i = 1,2$$

where subscript 0 signifies the steady state value of the variable and δ denotes a small perturbation about the steady state value. In the steady state, Eqs. 1 to 4 reduce to

$$9)\quad 0 = \left(\frac{\rho_{i0} - \beta}{\ell_i}\right) n_{i0} + \lambda_i c_{i0} + \frac{D_{2i}\delta_i^1}{\ell_{(i+1)}} n_{(i+1)0} + \frac{D_{1i}}{\ell_{(i-1)}}\delta_i^2\, n_{(i-1)0}$$

$$10)\quad 0 = \frac{\beta}{\ell_i} n_{i0} - \lambda_i c_{i0}$$

$$11)\quad 0 = K_i n_{i0} - a_i T_{i0}$$

$$12)\ \rho_{i0} = \rho_{ie0} - \alpha_i T_{i0} \qquad i = 1,2$$

Combining Eqs. 9-10 gives

$$13)\quad 0 = \frac{\rho_{i0}}{\ell_i}\, n_{i0} + \frac{D_{2i}\,\delta_i^1}{\ell_{(i+1)}}\, n_{(i+1)0} + \frac{D_{1i}}{\ell_i}\,\delta_i^2\, n_{(i-1)0}$$

where a value for ρ_{i0} can be found.

For core 1, i=1, Eq. 13 becomes

$$14)\quad 0 = \frac{\rho_{10} n_{10}}{\ell_1} + \frac{D_{21}}{\ell_2}\, n_{20}$$

and for core 2 , i=2, Eq. 13 reduces to

$$15)\quad 0 = \frac{\rho_{20} n_{20}}{\ell_2} + \frac{D_{12} n_{10}}{\ell_1}$$

Since both cores are identical and operating at the same power level, Eqs. 14 and 15 reduce to

$$16)\quad \rho_{i0} = -D = -D_{12} = -D_{21}$$

Substituting the relationships derived in Eqs. 9 and 16 into Eq. 5 and neglecting the nonlinear term ($\delta\rho_i \delta n_i$), Eq. 5 becomes

$$17)\quad \delta\dot{n}_i = -\frac{(D+\beta)}{\ell_i}\,\delta n_i + \lambda_i \delta c_i + \frac{D_{2i}}{\ell_{(i+1)}}\,\delta_i^1\,\delta n_{(i+1)} + $$
$$+ \frac{D_{1i}}{\ell_{(i-1)}}\,\delta_i^2\,\delta n_{(i-1)} + \frac{n_{i0}}{\ell_i}\,\delta\rho_i$$

and in the same manner using Eqs. 10 to 13, Eqs. 6 to 8 become

$$18)\quad \delta\dot{c}_i = \frac{\beta}{\ell_i}\,\delta n_i - \lambda_i\,\delta c_i$$

$$19)\quad \delta\dot{T}_i = K_i \delta n_i - a_i \delta T_i$$

20) $\delta\rho_i = \delta\rho_{ie} - \alpha_i \delta T_i$ $\qquad i=1,2$

Equations 17 through 20 represent the linearized coupled core one delayed neutron group point reactor model with linear independent temperature reactivity feedback.

In order to complete the linear model, the controller dynamics must be included. External reactivity, ρ_e, is applied to the system by the controller through control rod movement and/or reflector rotation. In this investigation it is assumed that the controller transfer function is of the form

$$21)\quad \frac{\delta\rho_{e(s)}}{u(s)} = \frac{K}{s^2 + 2\xi w_n s + w_n^2} = \frac{32}{s^2 + 8s + 32}$$

where

K is the gain,

ξ is the damping ratio (.707) of servo system,

$f_n = \omega_n/2\pi$ is the undamped natural resonant frequency (.9) of servo system, cycles/sec.

u is the control

s is the Laplace transform variable

A quadratic controller was chosen because it is a good approximation to some of the practical servo systems now employed in reactor control systems. Further a quadratic form of this type will in general closely approximate the behavior of most physical equipment (electric motor, hydraulic system, etc.) which might be utilized to drive a control rod or rotate a reflector.

In order to apply the state variable feedback concept, Eq. 21 must be written as a set of first-order differential equations. This can easily be accomplished by writing Eq. 21 in differential equation form

22) $\ddot{\delta\rho}_e + 8\dot{\delta\rho}_e + 32\,\delta\rho_e = 32\,u.$

Using phase variable notation, Eq. 22 is put in the form

23) $\dot{\delta\rho}_e = x_8$

and

24) $\dot{x}_8 = -\,32\,\delta\rho_e - 8\,x_8 + 32\,u.$

CHAPTER III

STATE VARIABLE FEEDBACK CONTROL

In applying the state variable concept to the design of a linear control system, the differential equations of the system are represented by a set of first-order vector-matrix differential equations of the form

25) $\dot{\underline{x}}\,(t) = \underline{A}\,\underline{x}\,(t) + \underline{b}u\,(t)$

26) $y\,(t) = \underline{c}^T\underline{x}(t)$

where

$\underline{x}$ (t) is an n-dimensional state vector

$\underline{A}$ is an n by n system matrix

$\underline{b}$ is an n dimensional control vector

n is the order of the system

$\underline{c}$ is the vector output

y(t) is the scalar system output

The superscript T denotes the transpose of the column vector and the dotted variables signify the time derivative of that variable.

When all the state variables are fed back through constant gain frequency independent elements, h_i, the control function, u, becomes

27) $u(t) = r(t) + \underline{h}^T\underline{x}(t)$

where

u(t) is the scalar control,

$\underline{h}$ is an n-dimensional vector which has as its elements the state variable feedback coefficients,

r(t) is an input variable.

Taking the Laplace transform of Eqs. 25 to 27,

28) $s\underline{x}(s) = \underline{A}\ \underline{x}(s) + \underline{b}\ u(s)$,

29) $y(s) = \underline{c}^T\ \underline{x}(s)$,

30) $u(s) = \underline{h}^T\ \underline{x}(s) + r(s)$

and substituting Eqs. 29-30 into Eq. 28, the closed-loop system transfer function is specified in terms of $\underline{h}$,

31) $$\frac{y(s)}{r(s)} = \underline{c}^T\ (s\ \underline{I} - \underline{A} - \underline{b}\ \underline{h}^T)^{-1}\ \underline{b}$$

where $\underline{I}$ is the identity matrix and the superscript, -1, indicates the inverse of the matrix.

Eq. 31 is written in more compact form by letting

32) $$(s\ \underline{I} - \underline{A} - \underline{b}\ \underline{h}^T) = \underline{F}$$

and

33) $$[F]^{-1} = \frac{[F_{ij}]^T}{\det F}$$

where

F_{ij} are the cofactors of the elements, f_{ij}, of the matrix, F

det F is the determinant of F.

The closed-loop system transfer function now becomes

34) $$\frac{y(s)}{r(s)} = \frac{\underline{c}^T [F_{ij}]^T \underline{b}}{\det F}$$

Eq. 31 provides a link to the classical design techniques since it is equivalent to

35) $$\frac{y(s)}{r(s)} = \frac{G(s)}{1 + G(s)\ H_{eq}(s)}$$

where

G(s) is the open-loop transfer function,

$$36)\quad G(s) = \frac{y(s)}{u(s)} = \underline{c}^T [s\, \underline{I} - \underline{A}]^{-1} \underline{b},$$

and $H_{eq}(s)$ is an equivalent feedback element,

$$37)\quad H_{eq}(s) = \frac{u(s)}{y(s)} = \frac{\underline{h}^T [s\, \underline{I} - \underline{A}]^{-1} \underline{b}}{\underline{c}^T [s\, \underline{I} - \underline{A}]^{-1} \underline{b}}$$

From the above equations it can be observed

a) The zeros of the open loop are also the zeros of the closed loop transfer function.

b) The location of the zeros in the closed loop transfer function is independent of the feedback coefficients and can only be altered by using a series compensator, or can be cancelled by a corresponding pole in $y(s)/r(s)$.

c) The poles of $H_{eq}(s)$ are the zeros of $G(s)$.

d) The closed-loop response or the desired system dynamics is a function of the feedback coefficients.

The state-variable feedback design technique consists of:

a) Specifying the desired dynamics of the system in terms of the closed-loop transfer function.

b) Formulating the closed-loop response in terms of $\underline{h}$, (Eq. 31), with $\underline{h}$ unspecified.

c) Solving a set of simultaneous linear algebraic equations in $\underline{h}$ which are generated by equating coefficients of powers of s of the denominator of Eq. 31 to coefficients of like powers of the desired closed-loop transfer function, the resulting values of $\underline{h}$ are the values of the feedback coefficients which will realize the desired system dynamics.

A digital computer program, based on the matrix formulation has been developed[3] for the design and analysis of state variable feedback systems. This program is designed to reduce the computational load which, although algebraic, becomes quite tedious for third-order systems or greater.

The program operates in the following manner:

a) Supplying the equations of the plant to be controlled in matrix notation ($\underline{A}$, $\underline{b}^T$, $\underline{c}^T$), the open-loop transfer function, Eq. 36

$$\frac{y(s)}{u(s)} = \underline{c}^T \left[s\ \underline{I} - \underline{A}\right]^{-1} \underline{b}$$

is calculated.

b) Knowing that the zeros of the open-loop are also the zeros of the closed-loop, unwanted zeros are removed by placing a corresponding pole in the desired transfer function which has already been specified.

c) Adding the poles of the desired transfer function to the previous input information, the program performs the calculations indicated in b) and c) on page 9.

This program can also obtain root locus information and perform feedback sensitivity studies. Because the program is based on a matrix approach, it maintains its generality. Another advantage of using the matrix approach is that all the transfer functions from the internal state variables to the input can be found by assigning appropriate values to the elements in the output vector $\underline{c}$.

CHAPTER IV

STATE VARIABLE FEEDBACK DESIGN OF REACTOR CONTROL SYSTEM

In order to illustrate the state variable feedback design technique, a control system is designed for a linearized coupled-core reactor.

Performing the indicated subscripted operations on Eqs. 5-to 8 and letting

$$\delta n_1 = x_1 \qquad \delta n_2 = x_4 \qquad \delta\rho_e = x_7$$

$$\delta c_1 = x_2 \qquad \delta c_2 = x_5$$

$$\delta T_1 = x_3 \qquad \delta T_2 = x_6$$

the equations describing the linearized coupled nuclear system with controller are written as

$$38)\quad \overset{\circ}{x}_1 = -\frac{(D+\beta)}{\ell_1} x_1 + \lambda_1 x_2 - \frac{\alpha_1 n_{10}}{\ell_1} x_3 + \frac{D}{\ell_2} x_4 + \frac{n_{10}}{\ell_1} x_7$$

$$39)\quad \overset{\circ}{x}_2 = \frac{\beta}{\ell_1} x_1 - \lambda_1 x_2$$

$$40)\quad \overset{\circ}{x}_3 = K_1 x_1 - a_1 x_3$$

$$41)\quad \overset{\circ}{x}_4 = \frac{D}{\ell_1} x_1 - \frac{(D+\beta)}{\ell_2} x_4 + \lambda_2 x_5 - \frac{\alpha_2 n_{20}}{\ell_2} x_6$$

$$42)\quad \overset{\circ}{x}_5 = \frac{\beta}{\ell_2} x_4 - \lambda_2 x_5$$

$$43)\quad \overset{\circ}{x}_6 = K_2 x_4 - a_2 x_6$$

$$44)\quad \overset{\circ}{x}_7 = x_8$$

$$45)\quad \overset{\circ}{x}_8 = -32\, x_7 - 8\, x_8 + 32\, u.$$

It is noted from the equations above that control is accomplished by perturbing core 1.

Eqs. 38 through 45 are put in the form of Eq. 25 where

46)
$$\underline{A} = \begin{bmatrix} -\dfrac{(D+\beta)}{\ell_1} & \lambda_1 & -\dfrac{\alpha_1 n_{10}}{\ell_1} & D/\ell_2 & 0 & 0 & \dfrac{n_{10}}{\ell_1} & 0 \\ \beta/\ell_1 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ K_1 & 0 & a_1 & 0 & 0 & 0 & 0 & 0 \\ D/\ell_1 & 0 & 0 & -\dfrac{(D+\beta)}{\ell_2} & \lambda_2 & -\dfrac{\alpha_2 n_{20}}{\ell_2} & 0 & 0 \\ 0 & 0 & 0 & \beta/\ell_2 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_2 & 0 & a_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -32 & -8 \end{bmatrix}$$

47) $\underline{b}^T = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 \end{vmatrix}$

Since the total incremental power is the output variable of interest, Eq. 26 becomes

48) $y = \underline{c}^T x = x_1 + x_4$

where

49) $\underline{c}^T = \begin{vmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{vmatrix}$

The feedback vector is given by

50) $\underline{h}^T = \begin{vmatrix} h_1 & h_2 & h_3 & h_4 & h_5 & h_6 & h_7 & h_8 \end{vmatrix}$

If the cores are identical and operating at the same steady state power level, then the parameters in core 1 (subscript 1) are equal to the corresponding parameters in core 2 (subscript 2). Assuming identical cores, consider a system having the following parameters:

n_0 = 320 megawatts; D = 0.00064

β = 0.0064; $K = 1.0 \dfrac{°R}{\text{Megawatt-sec}}$

ℓ = 3.2×10^{-5} seconds; $a = 1.0 \text{ sec}^{-1}$

λ = 0.1 seconds^{-1}; $\alpha = 0.8 \times 10^{-5}/°R$

Using these values, Eq. 46 becomes

$$\text{49)} \quad \underline{A} = \begin{bmatrix} -220 & .1 & -80 & 20 & 0 & 0 & 10^7 & 0 \\ 200 & -.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 20 & 0 & 0 & -220 & .1 & -80 & 0 & 0 \\ 0 & 0 & 0 & 200 & -.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -32 & -8 \end{bmatrix}$$

From the above equations, Eq. 32,

$\underline{F} = [S\,\underline{I} - \underline{A} - \underline{b}\underline{h}^T]$,

can be formulated;

$$\text{50)} \quad \underline{F} = \begin{bmatrix} (s+220) & -.1 & 80 & -20 & 0 & 0 & -10^7 & 0 \\ -200 & (s+.1) & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & (s+1) & 0 & 0 & 0 & 0 & 0 \\ -20 & 0 & 0 & (s+220) & -.1 & 80 & 0 & 0 \\ 0 & 0 & 0 & -200 & (s+.1) & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & (s+1) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & s & -1 \\ -32h_1 & -32h_2 & -32h_3 & -32h_4 & -32h_5 & -32h_6 & -32h_7 & (s+8-32h_8) \end{bmatrix}$$

Eq. 34, the closed-loop system transfer function specified in terms of h, now becomes

$$51) \quad \frac{y(s)}{r(s)} = \frac{32(F_{81} + F_{84})}{\det F}$$

The only limitation of this design technique, aside from the usual problem of physical realization, is that the pole-zero excess of the desired system cannot be less than that of the uncontrolled system. The coupled-core reactor has 5 zeros and 8 poles in the open-loop system; therefore, any synthesized system must have a pole-zero excess of at least 3.

In specifying the performance of the reactor control system, the following characteristics were considered:

a) a stable transient and steady-state response,

b) a short settling time; that is, the time required for the oscillation to die down to the specified absolute percentage of the final value and thereafter remain less than this value,

c) good steady-state sensitivity; that is, defined limits within which the control system will permit the reactor or its components to drift before corrective action occurs,

d) a fast response, and

e) the maximum amount of overshoot in the power level should last for only a maximum prescribed time.

Considering these characteristics, essentially second-order dynamics with a damping ratio of .707 and a fast response time was specified by placing a pair of dominant poles at $s = -10 \pm j\,10$ and another pole further out at $s = -200$. Because of its location, the pole at -200 has little effect on the system dynamics. However, it is necessary to satisfy the pole-zero excess condition. The damping ratio of .707 provides a step response with little overshoot and a very short settling time.

The system was further constrained by specifying a zero velocity error condition. A system with zero velocity error is one whose response to a ramp input follows the ramp input in the steady-state with no time lag. This condition was imposed by satisfying the following relationship:[4]

$$52)\quad \sum \frac{1}{P_{C.L.}} = \sum \frac{1}{Z_{C.L.}}$$

where

$\sum$ denotes summation

$P_{C.L.}$ refers to closed loop poles

$Z_{C.L.}$ refers to closed loop zeros

Since no zeros were specified in the desired transfer function, zero velocity error can be achieved by either retaining one of the inherent system zeros or by using series compensation. Because series compensation would increase the order of the system, one of the system zeros was retained.

From the state variable feedback program, the numerator of the open-loop transfer function was found to be

$$53)\quad \frac{32(F_{81}+F_{84})}{\det F} = 3.2 \times 10^{8}(s+.099)(s+1)(s+.0386)(s+1.313)(s+239.75)$$

Retaining the zero at $s = -.0386$ and cancelling the others, the denominator of the desired transfer function becomes

$$54)\quad \det F = (s+.099)(s+1)(s+1.313)(s+239.75)(s+200)(s^2+20s+200)(s+P_{C.L.})$$

where $P_{C.L.}$ is the location of the pole which satisfies the zero velocity error condition, Eq. 52 .

From Eqs. 53 and 54, the desired system transfer function becomes

$$55)\quad \left[\frac{y(s)}{r(s)}\right]_d = \frac{32(F_{81}+F_{84})}{\det F} = \frac{3.2 \times 10^8\ (s+.0386)}{(s+200)(s^2+20s+200)(s+P_{C.L.})}$$

Substituting the zero and pole locations into Eq. 52 yields

$$56)\quad P_{C.L.} = .036$$

Eq. 55 now becomes

$$57)\quad \left[\frac{y(s)}{r(s)}\right]_d = \frac{3.2 \times 10^8 (s+.0386)}{(s+.036)(s+200)(s^2+20s+200)}$$

The values of h_i's which realize the desired system dynamics are

$h_1 = -3.3993 \times 10^{-5}$

$h_2 = -4.2877 \times 10^{-8}$

$h_3 = +3.0239 \times 10^{-5}$

$h_4 = -3.39975 \times 10^{-5}$

$h_5 = -4.81369 \times 10^{-8}$

$h_6 = +3.05848 \times 10^{-5}$

$h_7 = -3.37513$

$h_8 = -.375$

Using these values, the control law stated by Eq. 27 becomes

$$58)\quad u = r - 3.3993 \times 10^{-5} x_1 - 4.2877 \times 10^{-8} x_2 + 3.0239 \times 10^{-5} x_3 - 3.39975 \times 10^{-5} x_4 - 4.81369 \times 10^{-8} x_5 + 3.05848 \times 10^{-5} x_6 - 3.37513\ x_7 - .375\ x_8$$

The control law is realized by measuring and summing all the state variables after each has been multiplied by the appropriate gain constant.

So far nothing has been said about the accessibility of the state variables. It has been assumed that all the state variables are available for measurement. This is not always the case. For instance, when delayed neutrons are included in the reactor dynamics, the precursor density, c, cannot be measured. These state variables can be generated from their describing equations. For example, Eq. 39, describing precursor density, can be written in the frequency domain as

$$59)\quad \frac{x_2}{x_1} = \frac{\beta}{\ell(s+\lambda)},$$

The state variable, x_2, can be generated by using the lag network described in Eq. 59.

Although the controlled variable, the total incremental power, x_1+x_4, is responding in a desired prescribed manner, some of the internal state variables may be behaving in an undesirable fashion. Therefore, it is necessary to find the response of all the internal state variables to an input, r(t). This is easily done by changing the elements in the output vector, $\underline{c}$. The closed-loop response has been determined for the following state variables:

$$60)\quad \frac{\delta n_1}{r} = \frac{3.2 \times 10^8 \;(s+.0346)(s+1.34)(s+219.73)}{(s+.0386)(s+1.31)(s+200)(s+239.75)(s^2+20s+200)}$$

$$61)\quad \frac{\delta c_1}{r} = \frac{6.4 \times 10^{10}(s+.0339)(s+1.34)(s+219.73)}{(s+.0386)(s+.0996)(s+1.31)(s+200)(s+239.75)(s^2+20s+200)}$$

$$62)\quad \frac{\delta T_1}{r} = \frac{3.2 \times 10^8 \;(s+.0339)(s+1.34)(s+219.73)}{(s+.0386)(s+1)(s+1.31)(s+200)(s+239.75)(s^2+20s+200)}$$

63) $$\frac{\delta n_2}{r} = \frac{6.4 \times 10^9 \ (s+.0872)(s+.986)(s+1.013)(s+1.132)}{(s+.0386)(s+.099)(s+1)(s+1.313)(s+200)(s+239.75)(s^2+20s+200)}$$

64) $$\frac{\delta c_2}{r} = \frac{1.28 \times 10^{12} \ (s+1)}{(s+.0386)(s+1.313)(s+200)(s+239.75)(s^2+20s+200)}$$

65) $$\frac{\delta T_2}{r} = \frac{6.4 \times 10^9 \ (s+.099 \pm 6.468 \times 10^{-4}_{j})}{(s+.0386)(s+.0996)(s+1.313)(s+200)(s+239.75)(s^2+20s+200)}$$

66) $$\frac{\delta \rho_e}{r} = \frac{32(s+.029)(s+199.69)}{(s+.0996)(s+1)(s+200)(s^2+20s+200)}$$

CHAPTER V

THE NONLINEAR PROBLEM

The purpose of this section is to determine the range of validity of the design technique when the control system that was derived for the linear model is applied to the nonlinear coupled-core system. This is accomplished by selecting various equilibrium power levels and subjecting them to a range of step inputs of power demand. The system behavior is studied using a time response digital program based on the 4th order Runge-Kutta method.

The equations describing the nonlinear coupled-core point reactor model are

67) $\mathring{x}_1 = -\frac{(D+\beta)}{\ell} x_1 + \lambda x_2 + \frac{D}{\ell} x_4 - \frac{\alpha}{\ell} x_1 x_3 + \frac{x_1 x_7}{\ell}$,

68) $\mathring{x}_2 = \frac{\beta}{\ell} x_1 - \lambda x_2$,

69) $\mathring{x}_3 = K x_1 - a x_3$,

70) $\mathring{x}_4 = \frac{D}{\ell} x_1 - \frac{(D + \beta)}{\ell} x_4 + \lambda x_5 - \frac{\alpha}{\ell} x_4 x_6$,

71) $\mathring{x}_5 = \frac{\beta}{\ell} x_4 - \lambda x_5$,

72) $\mathring{x}_6 = K x_4 - a x_6$,

73) $\mathring{x}_7 = x_8$

74) $\mathring{x}_8 = -32\, x_7 - 8\, x_8 + 32u$,

75) $u = r + \underline{h}^T \underline{x}$

where all the symbols are as previously defined except

$x_1 = n_1$ $\qquad x_4 = n_2$ $\qquad x_7 = \rho_e$

$x_2 = c_1$ $\qquad x_5 = c_2$

$x_3 = T_1$ $\qquad x_6 = T_2$

These equations are generated by combining Eqs. 1 to 4 (recalling that $\rho_{2e} = 0$ since only one core is being controlled) with the controller dynamics and the control law derived earlier.

The first equilibrium power level analyzed is that for which the control system was designed, namely 320 megawatts, x_{10}. Using the same parameter values that were outlined for the linear model, equilibrium values can be found for the system variables as follows from Eqs. 67 to 75.

76) $0 = -\frac{(D+\beta)}{\ell} x_{10} + \lambda x_{20} - \frac{D}{\ell} x_{40} - \frac{\alpha}{\ell} x_{10}x_{40} + \frac{x_{10}x_{70}}{\ell}$

77) $0 = \frac{\beta}{\ell} x_{10} - \lambda x_{20}$

78) $0 = Kx_{10} - ax_{30}$

79) $0 = \frac{D}{\ell} x_{10} - \frac{(D+\beta)}{\ell} x_{40} + \lambda x_{50} - \frac{\alpha}{\ell} x_{40}x_{60}$

80) $0 = \frac{\beta}{\ell} x_{40} - \lambda x_{50}$

81) $0 = Kx_{40} - ax_{60}$

82) $0 = x_{80}$

83) $0 = -32\, x_{70} - 8x_{80} + 32\, u_0$

84) $u_0 = r_0 - 3.3993 \times 10^{-5}\, x_{10} - 4.2877 \times 10^{-8}\, x_{20} +$

$+ 3.0239 \times 10^{-5}\, x_{30} - 3.39975 \times 10^{-5}\, x_{40} -$

$- 4.81369 \times 10^{-8}\, x_{50} + 3.05848 \times 10^{-5} x_{60} -$

$- 3.37513\, x_{70} - .375\, x_{80}$

From Eq. 77, it can be seen that

$$85)\quad x_{20} = \frac{\beta}{\lambda \ell} x_{10} = 6.4 \times 10^5 \text{ megawatts}$$

Also, Eq. 78 yields

$$86)\quad x_{30} = \frac{K}{a} x_{10} = 320\ °R$$

The relationship of x_{50} and x_{60} with x_{40} can be determined from Eqs. 80 and 81.

$$87)\quad x_{50} = \frac{\beta}{\lambda \ell} x_{40}$$

and

$$88)\quad x_{60} = \frac{K}{a} x_{40}$$

Substituting these relationships into Eq. 79, gives

$$89)\quad 0 = \frac{D}{\ell} x_{10} - \frac{D}{\ell} x_{40} - \frac{\alpha K}{a \ell} x_{40}^2$$

Rearranging, Eq. 89 becomes

$$90)\quad x_{40}^2 + \frac{aD}{\alpha K} x_{40} - \frac{aD}{\alpha K} x_{10} = 0$$

Since x_{10} is a constant, x_{40} can be determined as follows:

$$91)\quad x_{40} = -\left(\frac{aD}{2\alpha K}\right) \pm \sqrt{\left(\frac{aD}{2\alpha K}\right)^2 + \frac{aD}{\alpha K} x_{10}}$$

x_{40} is the incremental power in core 2, therefore, x_{40} cannot be negative. Since all the parameters in Eq. 91 are positive, the correct relationship for x_{40} is

$$92)\quad x_{40} = -\left(\frac{aD}{2\alpha K}\right) + \sqrt{\left(\frac{aD}{2\alpha K}\right)^2 + \frac{aD}{\alpha K} x_{10}}$$

Substituting numerical values, x_{40} is found to be 124.92 Megawatts. Now, x_{50} and x_{60} can easily be determined from Eqs. 87 and 88. These values are

$$x_{50} = 249.84 \times 10^3 \text{ Megawatts}$$

and

$$x_{60} = 124.92\ ^\circ R$$

Eq. 82 tells us that

$$x_{80} = 0$$

From Eq. 76, x_{70} can be found in terms of the other system variables.

$$93)\quad x_{70} = D - D\frac{x_{40}}{x_{10}} + \alpha x_{30}$$

Since x_{10}, x_{30}, and x_{40} are known, x_{70} can be evaluated.

$$x_{70} = 2.95 \times 10^{-3}$$

All that remains to be done is to determine r_0, the initial condition for the step input variable.

From Eq. 83, it is noted that

$$94)\quad x_{70} = u_0$$

Since the equilibrium value of all the system variables have been determined, Eq. 84 yields the result

$$r_0 = .054.$$

The other equilibrium power levels examined are 640 Megawatts and zero power (100 watts). The value of the system variables for equilibrium operation are determined from the same relationships derived on the preceding pages. Table 1 shows the value of the system variables for various equilibrium power levels.

TABLE I. EQUILIBRIUM STATE VARIABLES FOR POWER LEVELS INVESTIGATED

State Variable	Zero Power	Linear Design Power	Twice Linear Design Power
n_{10}	10^{-4} MW	320 MW	640 MW
C_{10}	.2 MW	6.4×10^5 MW	1.28×10^6 MW
T_{10}	10^{-4} °R	320 °R	640 °R
n_{20}	10^{-4} MW	124.92 MW	189.78 MW
C_{20}	.2 MW	2.4984×10^5 MW	3.7956×10^5 MW
T_{20}	10^{-4} °R	124.92 °R	189.78 °R
ρ_{e0}	$.8 \times 10^{-5}$	2.95×10^{-3}	5.57×10^{-3}
$\dot{\rho}_{e0}$	0	0	0
r_0	$\simeq 0$	.054	.100

where MW denotes megawatts and °R, degrees Rankine.

CHAPTER VI

SIMULATION STUDIES OF THE LINEAR AND NONLINEAR SYNTHESIZED SYSTEM

In this chapter, the simulation studies of the linear and nonlinear synthesized system are discussed.

The system to be controlled consists of two identical reactors, some finite distance apart, operating at 320 Megawatts. When a step demand in power is placed in the system the power starts to rise quickly in core 1. The rate of production of delayed neutrons catches up to the initial power change and slows it down. Finally, the temperature reactivity feedback levels the power off at a new steady state value. Correspondingly an increase in power level in core 2 is caused by the coupling between the cores. Fig. 1 shows the incremental power response of the linear system to a step demand in power. It is exactly the response that was specified.

Fig. 2 shows the amount of externally induced reactivity, x_7, (37¢) that is required to almost double the initial steady-state power level and the temperature response of each core in the linear system to a step demand of power. Fig. 3 represents the short-time temperature response to a step demand in power. The temperatures are proportional to their corresponding power levels.

Fig. 4 shows the response of the individual cores, as the cores are moved together. This corresponds to increasing the coupling coefficient, D, between the two cores. It can be seen that the power in the controlled core, core 1, decreases, and the power in the other core increases while the total response remains the same. Finally, at D = .0384, there is almost an equal sharing of power. For coupling coefficients greater than D = .0384 the power in both cores blows up. In effect this corresponds to a physical situation where the cores coalesce and indicates that the limits of the validity of the equations describing the system have been exceeded.

Fig. 5 shows the sensitivity of λ, the effective delayed neutron precursor decay constant, on the overall system response. The dashed line represents the desired response. In order to maintain the desired response λ can't be varied more than $\pm$ 10%.

A root locus plot of the synthesized linear system as a function of controller gain is given in Fig. 6. It is clear that the system will not go unstable for any value of controller gain.

Fig. 7 shows the dependence of the system transfer function on the equilibrium power level, n_0. From Fig. 7, it can be seen that increases in the steady-state power level, n_0, decrease the damping of the system response. The steady-state power level can be raised to approximately 960 Megawatts before the system starts to oscillate and becomes unstable. Decreasing n_0 results in a more heavily damped system response.

Fig. 8 shows the response of the linear system to a ramp input. The power ramp lags the input ramp by approximately 0.16 secs.

Variations in the feedback coefficients (h_4, h_5 and h_6) associated with the power, precursor concentration, and temperature of the uncontrolled core respectively, have very little affect on the system dynamics and neglecting them only slightly changes the system response. This is as expected since the state variables they are feeding back are very small compared to their respective quantities in the controlled core. The system response is very sensitive to changes in h_7 and h_8, corresponding to control rod position and velocity respectively, because they are quite large. The final steady-state power level is affected by variations in the power feedback coefficients, h_1 and h_4. The power level increases for decreasing values of h. This affect is more readily seen in variations of h_1 since the power in core 1 is a lot larger than the power in the uncontrolled core. The feedback coefficients, h_2 and h_3, corresponding to the precursor concentration and temperature of the controlled core can be varied $\pm$ 15% without appreciably changing the system response.

Fig. 9 shows the response of the non-linear system, operating at the designed 320 MW, to various step demands in power. The desired system response is obtained for step demands in power up to 35% of the equilibrium value. There is only a slight change in the system dynamics for step demands of power up to 50% of the steady-state power level. The step response for negative inputs is more heavily damped than the responses for corresponding positive inputs.

The next equilibrium point examined was 640 MW. Fig. 10 shows that system response for step demands in power up to 40% of the equilibrium show only a slight change in the system dynamics. The system oscillates sooner than for the designed 320 MW operating level. Once again, the responses to negative inputs are more heavily damped than for their corresponding positive inputs.

The final steady state level studied was that of zero power. For this condition the magnitude of all the state variable should be zero. However, since there is an equilibrium point at the origin, a small finite power (100 watts) was assumed in order to get a response from the system. Fig. 11 shows that the larger the step demands in power, the less damped the system becomes. For step demands in power larger than the ones shown, the response becomes oscillatory.

The sensitivity of the system response to parameter variations was examined at each of the non-zero steady state power levels. The results were essentially identical with those reported for the linearized system.

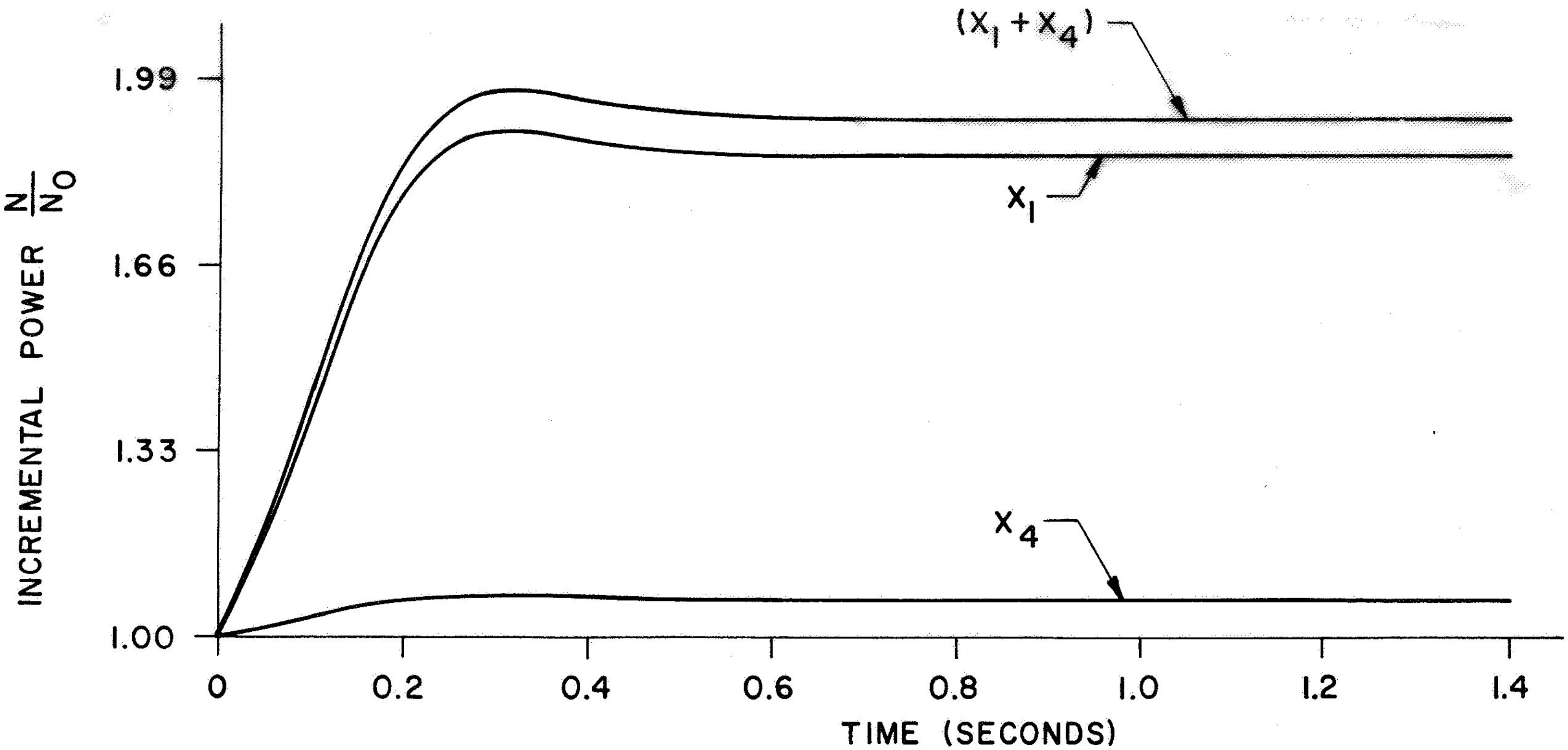

Fig. 1: Incremental Power Response of Linear System to a Step Demand in Power

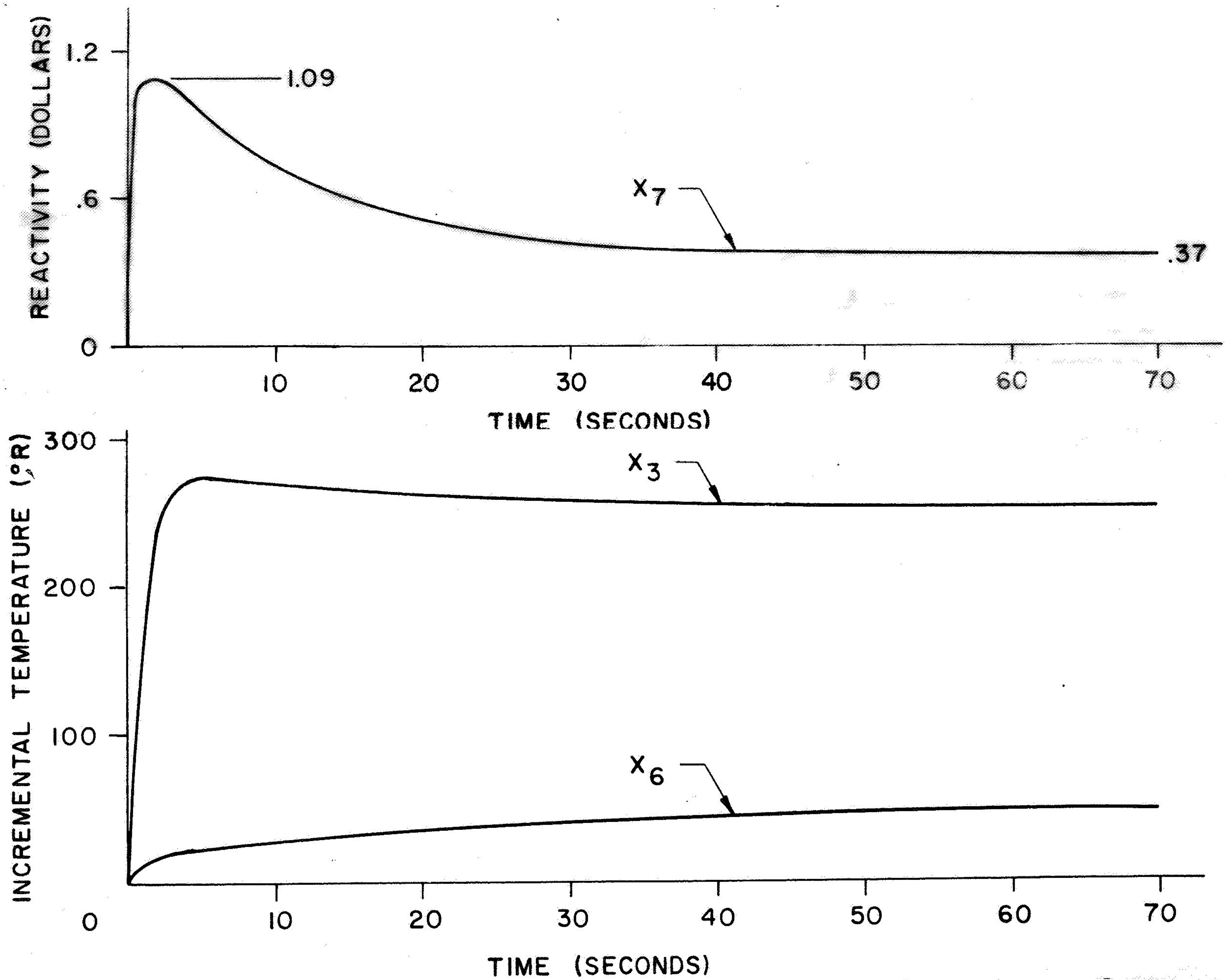

Fig. 2: **State Variable Response of Linear System to a Step Demand in Power**

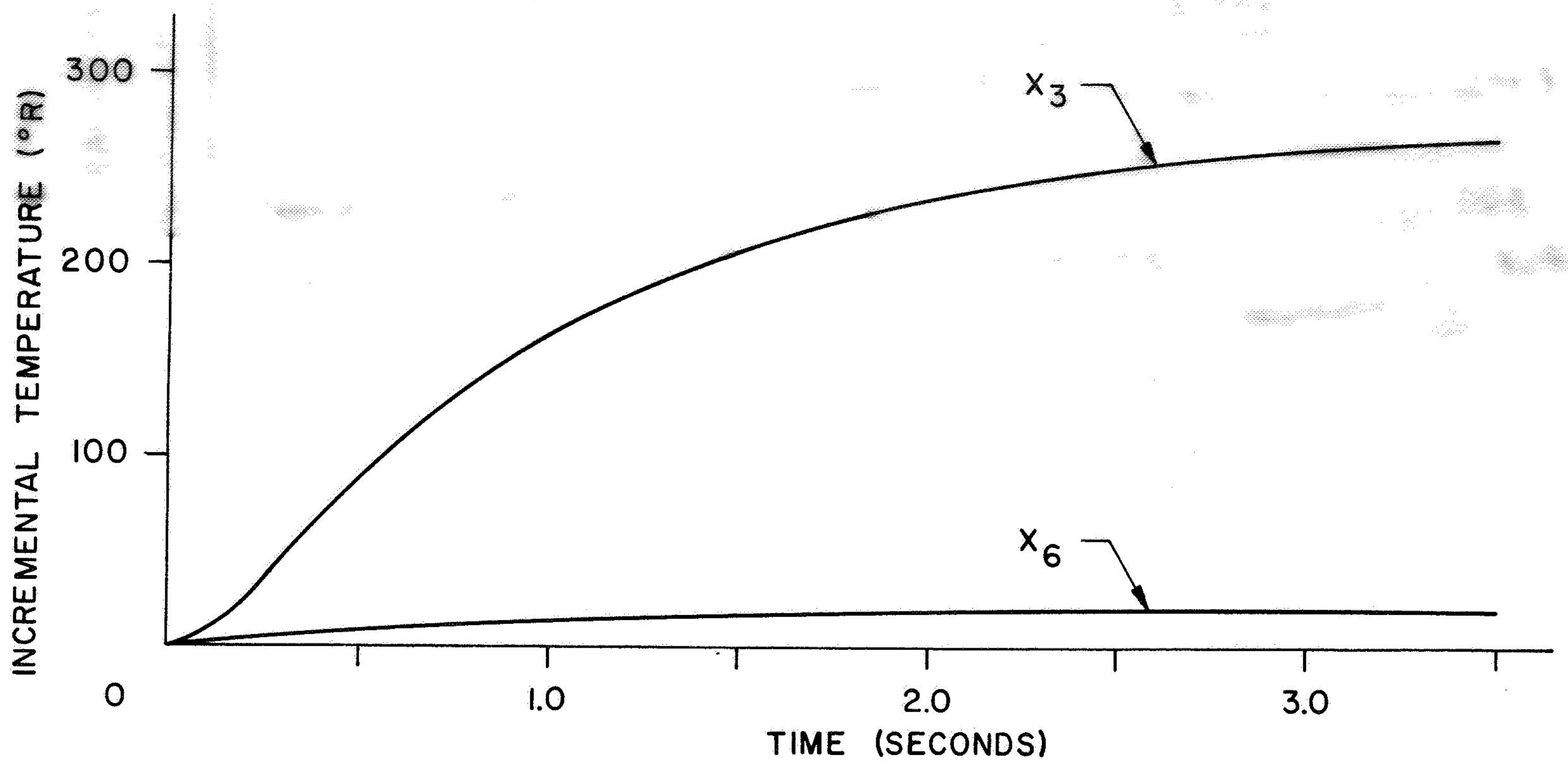

Fig. 3: Temperature Response to a Step Demand in Power

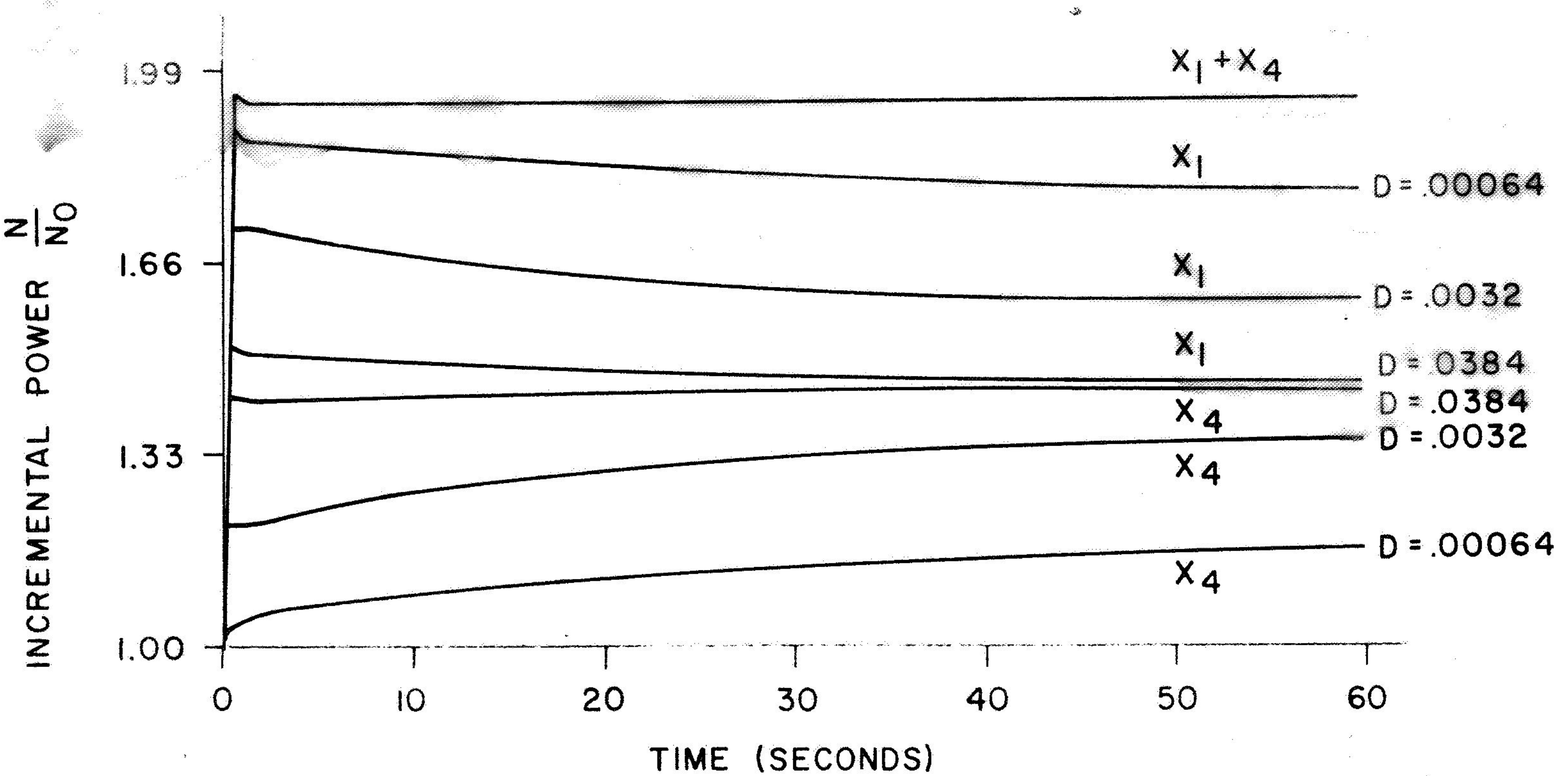

Fig. 4: Incremental Power Response of Linear System to a Step Demand in Power for Various Coupling Coefficients, D

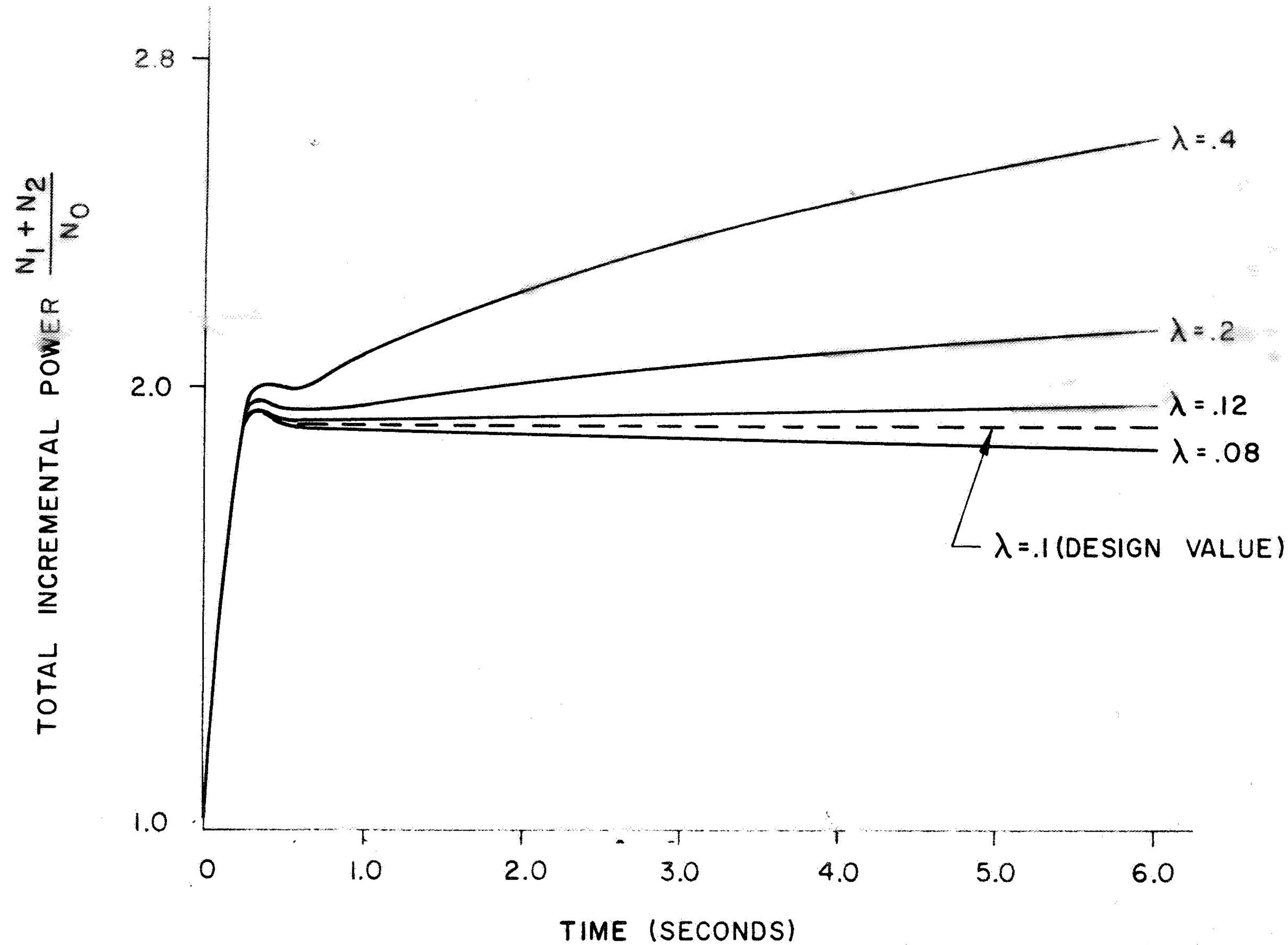

Fig. 5: Response of **Linear** System to a Step Demand in Power for Various Decay Constants, λ

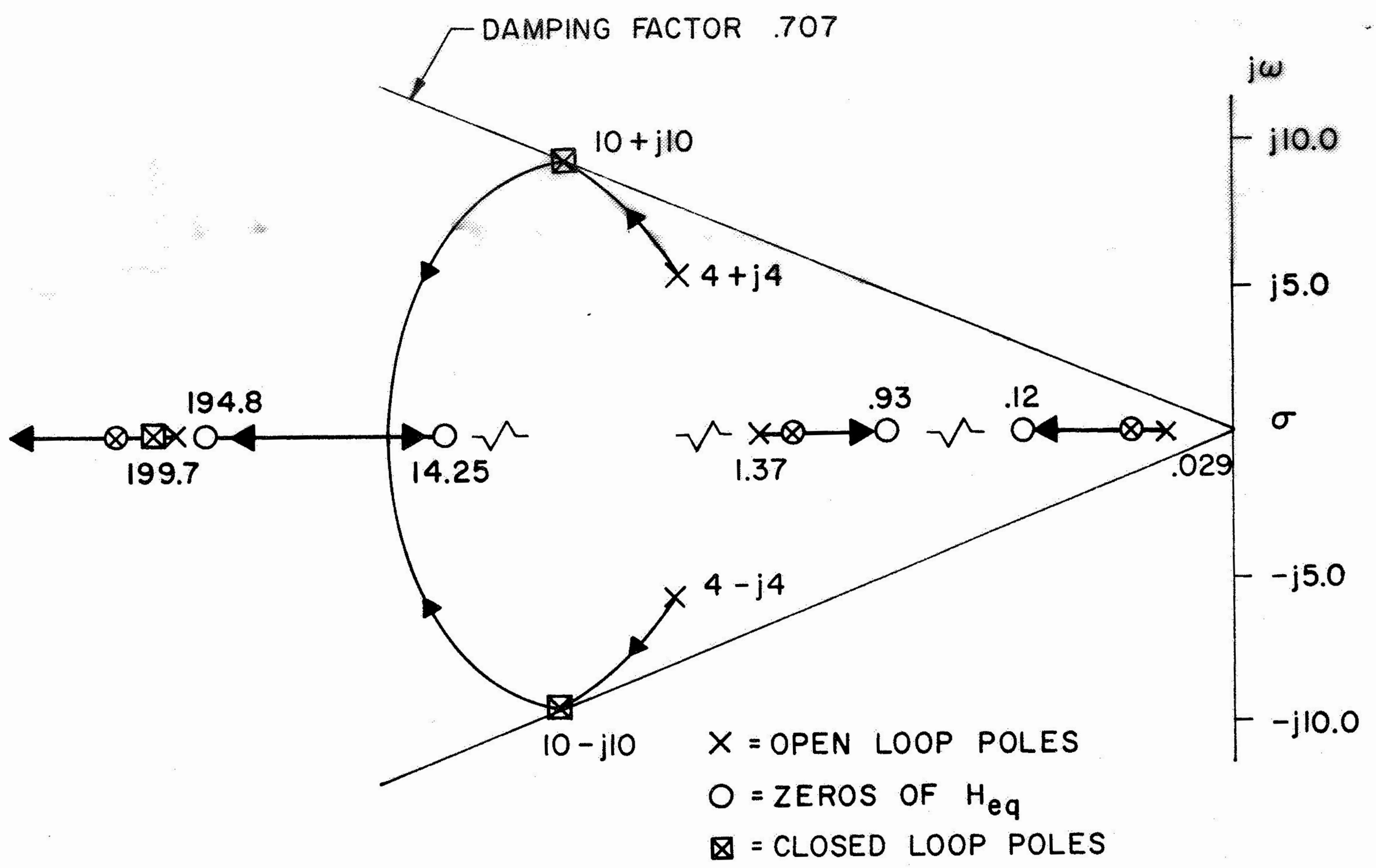

Fig. 6: Root Locus of Synthesized System

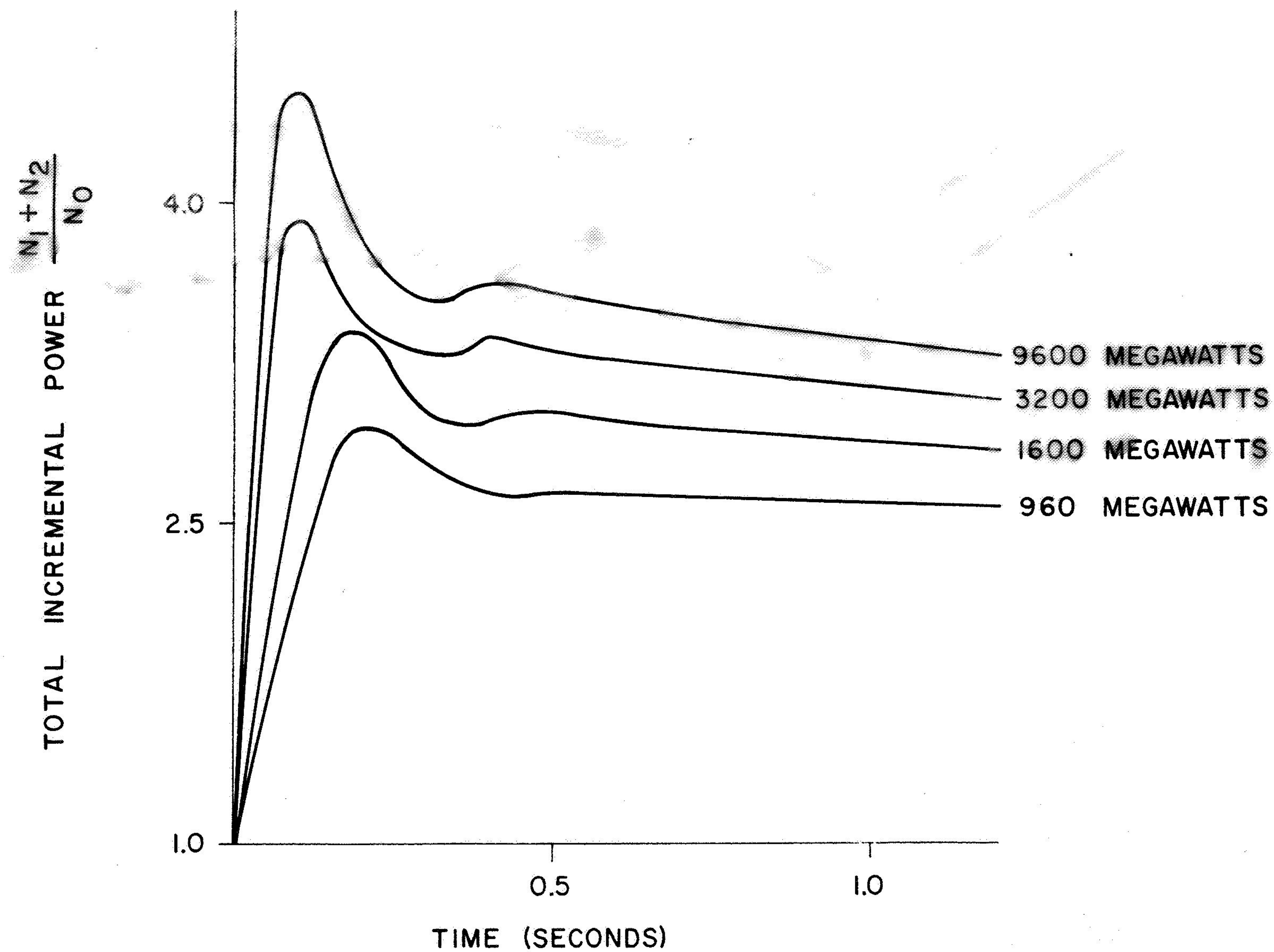

Fig. 7: Response of Linear System to a Step Demand in Power For Various Steady-State Power Levels, N_{oi}.

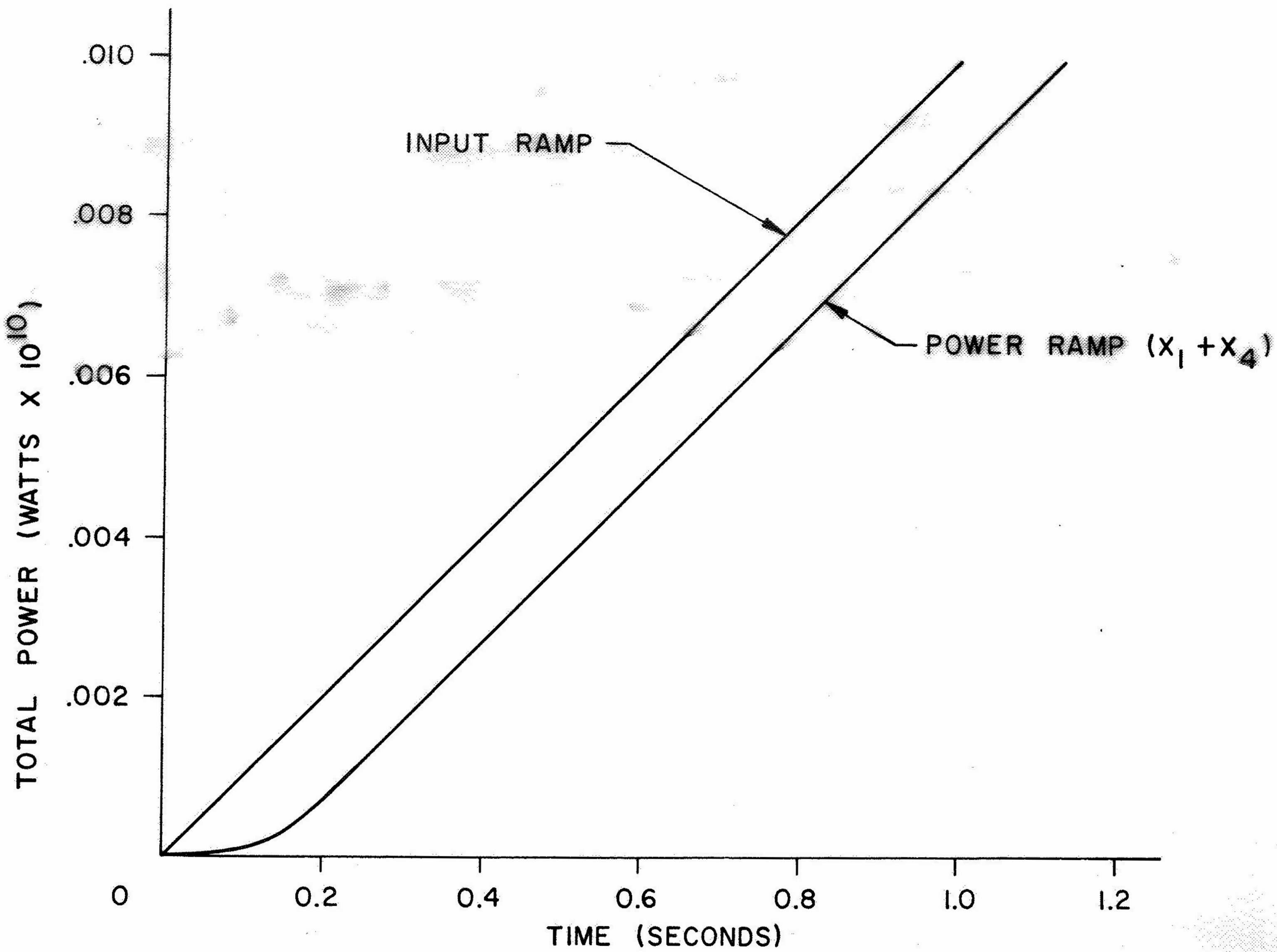

Fig. 8: Response of Linear System to a Ramp Demand in Power

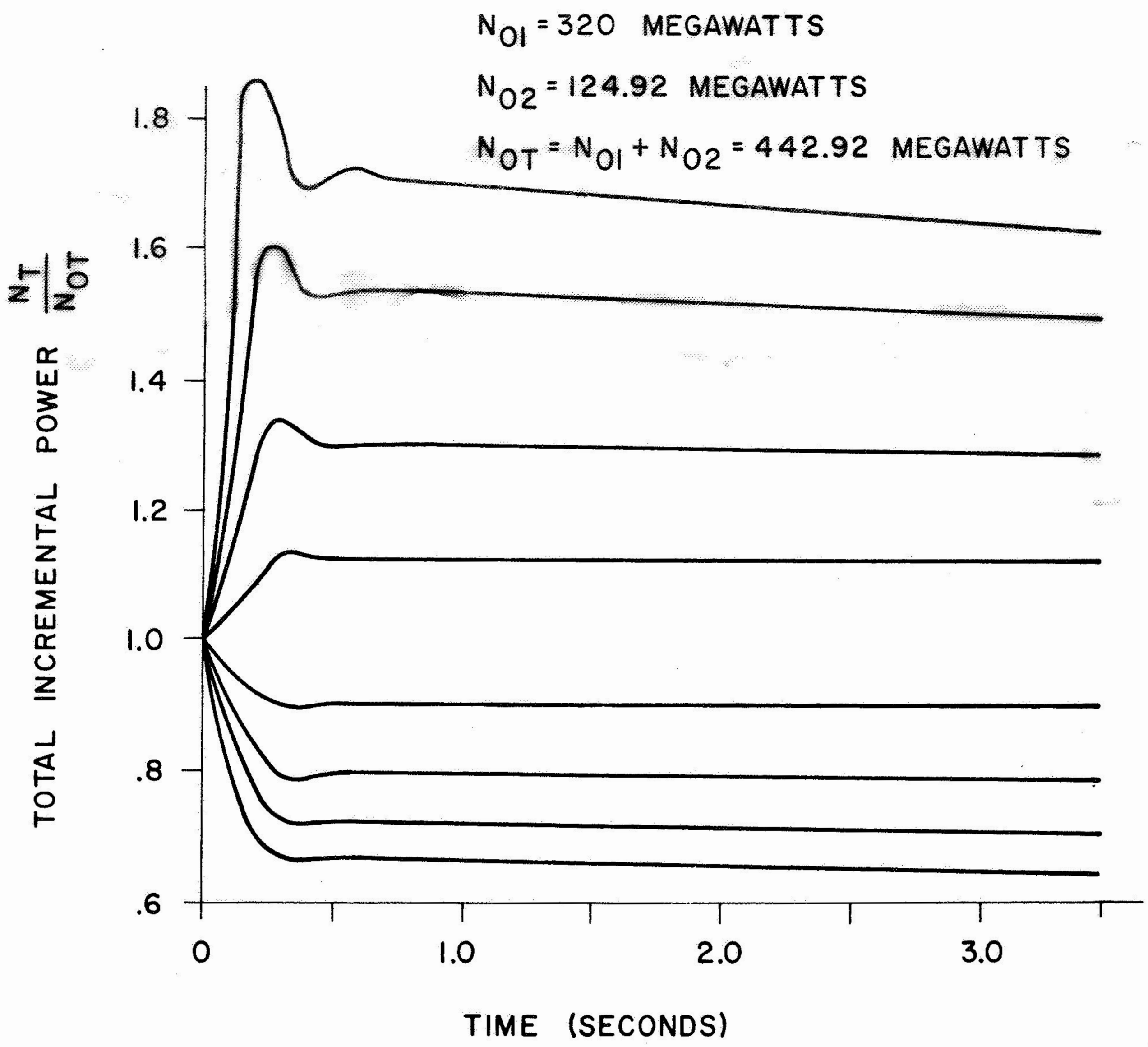

Fig. 9: Response of Coupled Core Reactor to a Step Demand in Power

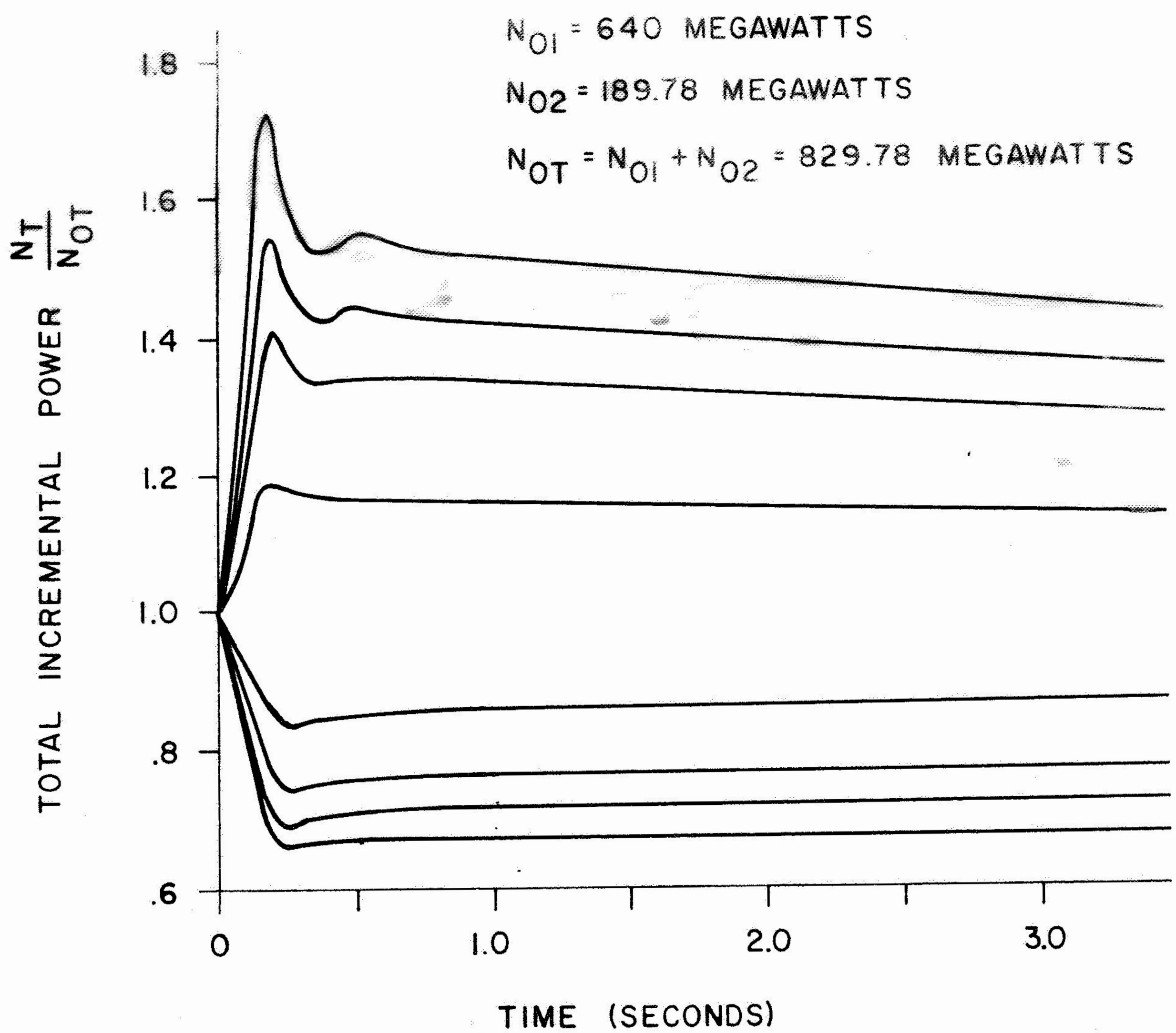

Fig. 10: Response of Coupled Core Reactor to a Step Demand in Power

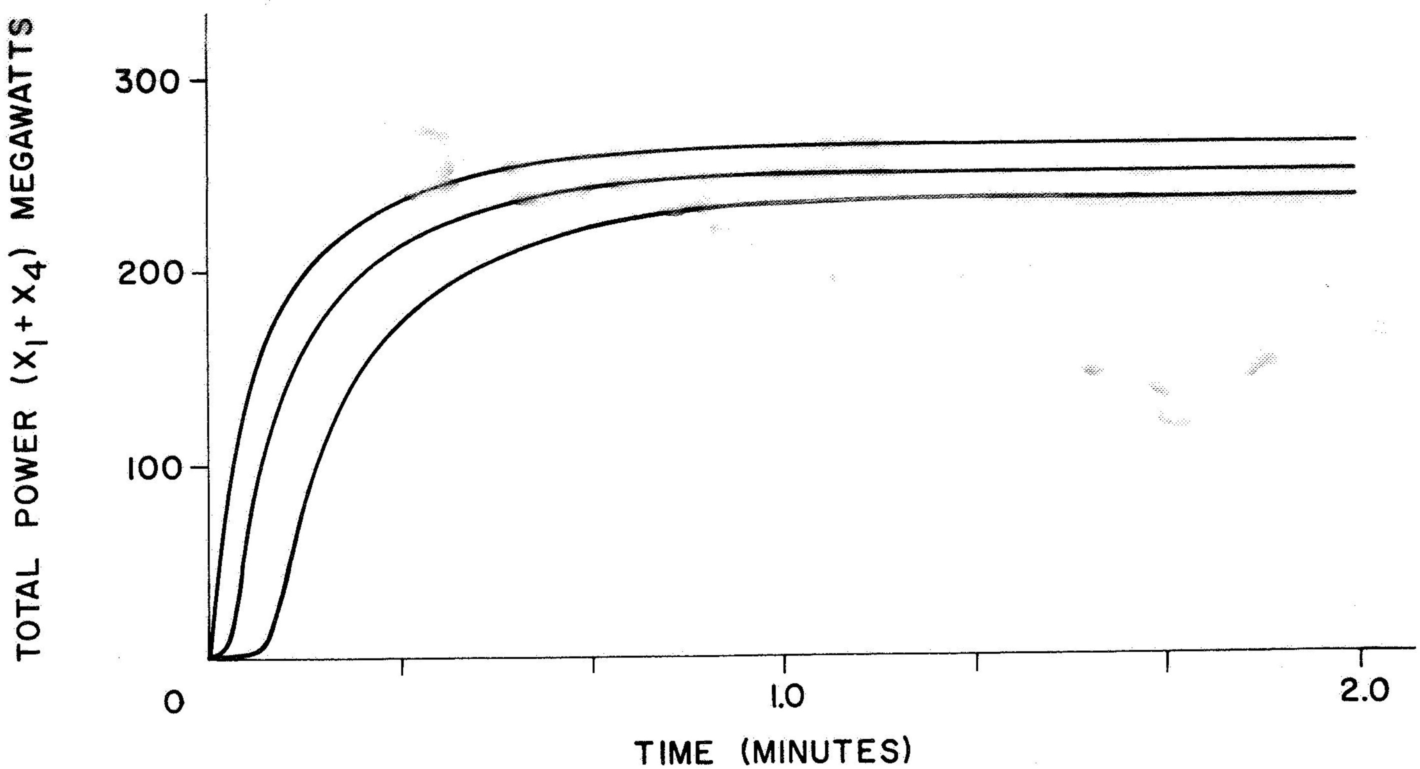

Fig. 11: Response of Zero-Power Coupled-Core Reactor to Step Demand in Power

CHAPTER VII

CONCLUSIONS

The simulation studies show that:

a) An effective control system for a coupled-core reactor can be designed using state variable feedback techniques;

b) The desired system dynamics, specified in terms of a closed-loop transfer function, are exactly realized by feeding back all the state variables through constant gain elements (feedback coefficients). As a consequence, almost any desired response can be imposed upon the system merely by determining the proper feedback coefficients;

c) Inaccessible state variables are not a problem. They can be generated from their describing equations and fed back through frequency dependent elements;

d) Conventional control specifications such as zero velocity error, damping ratio, overshoot, etc., can be employed in the design method. They can be realized with or without series compensation;

e) The control law derived for the linear system is applicable to the non-linear problem. The steady-state power level can almost be tripled before the system starts to oscillate;

f) The effective delayed neutron precursor decay constant, λ, cannot be varied more than $\pm$ 10% without distorting the desired response.

Finally, being formulated in matrix notation, the method maintains its generality. Another advantage of using the matrix approach is that all the internal state variable transfer functions can be determined merely by changing elements in the output vector $\underline{c}$. This is very important because one cannot design an effective control system without knowing how all the system variables are performing.

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